Quantitative Analysis:

Decision Making

1. Optimistic/Maximax criterion:

Example 1: From the given pay off table give decision according to maxi-max criteria.

Pay off Matrix

Courses of action	**Events**		
	Rainy	**Cloudy**	**Sunny**
S1	**500**	**500**	**500**
S2	**300**	**550**	**550**
S3	**100**	**350**	**600**

Solution:

The maximum payoff along the course of action S1 is 500
The maximum payoff along the course of action S2 is 550
The maximum payoff along the course of action S3 is 600

Pay off Matrix

Courses of action	**Events**			**Row maxima**
	Rainy	**Cloudy**	**Sunny**	
S1	500	500	500	500
S2	300	550	550	550
S3	100	350	600	**600 (Maximax)**

Decision: The decision maker selects the maximum among the maximum payoffs which is 600. Hence according to the maximax criterion, the decision maker selects strategy S3.

Pessimistic/maximin Criterion

Example 2: From the given pay off table give decision according to maxi-min criteria.

Pay off Matrix

Courses of action	**Events**		
	Rainy	**Cloudy**	**Sunny**
S1	**500**	**500**	**500**
S2	**300**	**550**	**550**
S3	**100**	**350**	**600**

Solution:

The minimum payoff along the course of action S1 is 500.
The minimum payoff along the course of action S2 is 300.
The minimum payoff along the course of action S3 is 100.

Pay off Matrix

Courses of action	**Events**			**Row minima**
	Rainy	**Cloudy**	**Sunny**	
S1	500	500	500	500 (Maximin)
S2	300	550	550	300
S3	100	350	600	100

Decision: The decision maker selects the maximum among the given minimum payoff in row minima, which is 500. Hence according to minimum payoff in row-minima, the decision makers select the course of action S1.

Example 3:
From the given payoff table, give decision according to maximax and maximin criteria.

State of nature	Strategy		
	S1	S2	S3
A	4	-2	7
B	0	6	3
C	-5	9	2
D	3	1	4

Minimax Regret/Opportunity Loss Criterion.

Regret: A regret is the difference between the actual payoff and the payoff one could have received if one knew which event was going to occur.

Question: From the given payoff table give the decision according to minimax regret criterion.

Payoff matrix

Course of action	State of nature(Events - Demand		
	Rainy	Cloudy	Sunny
S1	500	500	500
S2	300	550	550
S3	100	350	600

Solution:
The regret values of the given payoff matrix can be calculated as:
For rainy days: The maximum payoff is 500.
Regret for S1 and rainy = 500-500 = 0
Regret for S2 and rainy = 500-300 = 200
Regret for S2 and rainy = 500-100 = 400
For cloudy days: The maximum payoff is 550.
Regret for S1 and cloudy = 550-500 = 50
Regret for S2 and cloudy = 550-550 = 0
Regret for S2 and cloudy = 550-350 = 200
For sunny days: The maximum payoff is 600.
Regret for S1 and sunny = 600-500 = 100
Regret for S2 and sunny = 600-550 = 50
Regret for S2 and sunny = 600-600 = 0

Regret Payoff matrix

Course of action	State of nature(Events - Demand			Row maxima
	Rainy	Cloudy	Sunny	
S1	0	50	100	100=> minimax
S2	200	0	50	200
S3	400	200	0	400

Decision: The maximum regret is selected from each two and is placed in row maxima column. After that the minimum regret is selected among the selected maximum regrets which is 100 corresponding course of action S1. Hence according to minimax regret criterion course of action S1 is selected.

Question: Prepare the regret table from the given conditional profit table.

Demanded units	**Decision alternatives/Strategy/Course of Action**			
	15	**16**	**17**	**18**
15	**150**	**120**	**90**	**90**
16	**150**	**160**	**130**	**100**
17	**150**	**160**	**170**	**140**
18	**150**	**160**	**170**	**180**

Formula:
Let S = Stock, D = Demand
Case I
When Stock is less or equals to Demand $(S \leq D)$
Payoffs = Profit × Quantity sold
$P \times S$
Case II
When stock is more than demand (When demand is less than stock) D<S
Payoffs = Profit ×Quantity sold – Quantity unsold × CP
$= P \times D – (S - D) \times CP$

- Profit = SP – CP

Example: A man buys a bottle at 9 and sells at 11 each. His past experience shows that his daily selling is not less than 10 and not more than 12. All the bottle left over are worthless. Prepare payoff and regret table.
Solution:
CP = 9
Sp = 11
Profit = SP – CP = 11 – 9 = 2
Let S = Stock, D = Demand
Case I
When Stock is less or equals to Demand $(S \leq D)$
Payoffs = Profit × Quantity sold
$2 \times S = 2S$
Case II
When stock is more than demand (When demand is less than stock) D<S
Payoffs = Profit ×Quantity sold – Quantity unsold × CP
$= P \times D – (S - D) \times CP$
$= 2 \times D – (S - D) \times 9$
$= 2D – 9S + 9D$
$= 11D – 9S$
When
S = 10 , D = 10, Payoff = $2S = 2 \times 10 = 20$
S = 11, D = 10, Payoff = $11D – 9S = 110 – 99 = 11$
S = 12, D = 10, Payoff = $11D = 9S = 110 – 108 = 2$
When
S = 10 , D = 11, Payoff = $2S = 2 \times 10 = 20$
S = 11, D = 11, Payoff = $2S = 22$
S = 12, D = 11, Payoff = $11D = 9S = 121 – 108 = 13$
When
S = 10 , D = 12, Payoff = $2S = 2 \times 10 = 20$
S = 11, D = 12, Payoff = $2S = 22$
S = 12, D = 12, Payoff = $2S = 24$

Now, Payoff table

Demand / Stock	10	11	12
10	20	20	20
11	11	22	22
12	2	13	24

Again regret table:

Demand / Stock	10	11	12	Row maxima
10	0	2	4	4
11	9	0	2	9
12	18	9	0	18

Qn: The newspaper vendor buys the paper at Rs. 3 and sells them at 4. If the paper is not sold on the particular day, it has the salvage value of Rs. 1. The newspaper vendor knows that he cannot sell more than 50 and not less than 45. Construct a payoff and regret table.

Formula:

Let S = Stock, D = Demand

Case I

When Stock is less or equals to Demand $(S \leq D)$

Payoffs = Profit × Quantity sold

$P \times S$

Case II

When stock is more than demand (When demand is less than stock) $D<S$

Payoffs = Profit ×Quantity sold – Quantity unsold × (CP-Salvage value)

$= P \times D – (S - D) \times (CP-1)$

Expected Monetary Value (EMV) = Payoff × Probabilities

Expected profit with perfect Information (EPPI) = Diagonal Payoff × Probabilities

Expected Value with Perfect Information(EVPI) = EPPI – max EMV

Example: The Pukar Shop promises its custors to deliver within four hours on all flower orders. All flowers are purchased on the previos day and delivered to Pukar by 8:00 AM the next morning. Pukar daily demand for roses is as follows:

Dozens of roses	**7**	**8**	**9**	**10**
Probabilities	**0.1**	**0.2**	**0.4**	**0.3**

Pukar purchases roses for Rs. 10 per dozen and sells them for Rs. 30. All unsold roses are donated to a local hospital. How many dozens of roses should Pukar order each evening to maximize profit? What is the optimum expected profit?

Solution:

Selling Price (SP) = 30

Cost Price (CP) = 10

Profit = SP – CP = 30 -10 = 20

Case I

When Stock is less or equals to demand

Payoffs = Profit × Quantity sold = 20S

Case II

When Stock is more than demand

Payoffs = Profit × Quantity sold – Quantity unsold × CP

$= 20D – (S – D)10$

$= 20D – 10S + 10D$

$= 30D – 10S$

Now
When
S = 7 D = 7 Payoff = 20.7 = 140
S = 8 D = 7 Payoff = 30.7 – 10.8 = 210 – 80 = 130
S = 9 D = 7 Payoff = 30.7 – 10.9 = 210 – 90 = 120
S = 10 D = 7 Payoff = 30.7 – 10.10 = 210 – 100 = 110
When
S = 7 D = 8 Payoff = 20.7 = 140
S = 8 D = 8 Payoff = 20.8 = 160
S = 9 D = 8 Payoff = 30.8 – 10.9 = 240 – 90 = 150
S = 10 D = 8 Payoff = 30.8 – 10.10 = 240 – 100 = 140
When
S = 7 D = 9 Payoff = 20.7 = 140
S = 8 D = 9 Payoff = 20.8 = 160
S = 9 D = 9 Payoff = 20.9 = 180
S = 10 D =9 Payoff = 30.9 – 10.10 = 270 – 100 = 170
When
S = 7 D = 10 Payoff = 20.7 = 140
S = 8 D = 10 Payoff = 20.8 = 160
S = 9 D = 10 Payoff = 20.9 = 180
S = 10 D = 10 Payoff = 20.10 = 200
Now,
Tabulating payoffs,

Demand (States of nature) / **Stock**	**Payoffs**			
	0.1	**0.2**	**0.4**	**0.3**
	7	**8**	**9**	**10**
7	140	140	140	140
8	130	160	160	160
9	120	150	180	180
10	110	140	170	200

Calculation of EMV
EMV of Stock 7 = 140×0.1 + 140×0.2 + 140×0.4 + 140×0.3 = 140
EMV of Stock 8 = 130×0.1 + 160×0.2 + 160×0.4 + 160×0.3 = 157
EMV of Stock 9 = 120×0.1 + 160×0.2 + 180×0.4 + 180×0.3 = 168 (Highest EMV)
EMV of Stock 10 = 110×0.1 + 140×0.2 + 170×0.4 + 200×0.3 = 167

Decision: Since Alternative stock 9 has the highest EMV, So Pukar should order 9 dozens of roses each evening to maximize his profit. The optimal expected profit = Rs. 168

Calculation of EPPI

X (Diagonal Payoff)	**P (Probability)**	**XP**
140	0.1	14
160	0.2	32
180	0.4	72
200	0.3	60
		$\sum XP$ **= 178**

Therefore, EPPI = 178

Calculation of EVPI

EVPI = EPPI – Max. EMV => EVPI = 178 – 168 = 10