Model Question 2078

Grade 12 Mathematics

Set – 1

Group A **1 ×11=11**

1. The value of ${}^nP_r$ and ${}^nC_r$ will be equal when

   a. n=r
   b. r = $\frac{n}{2}$
   c. r = 1 or n
   d. r = 0 or 1

Ans:d

2. The condition for polynomial equation $ax^2 + bx + c = 0$ to be quadratic is

   a. a >0
   b. r = a < 0
   c. a ≠ 0
   d. b ≠ 0 or c ≠ 0

Ans:c

3. The general solution of sinx = $\frac{1}{\sqrt{2}}$ is

   a. $2n\pi + \frac{\pi}{4}$
   b. $(2n+1)\frac{\pi}{2}$
   c. $n\pi + \frac{\pi}{4}$
   d. $2n\pi + \frac{\pi}{2}$

Ans:a

4. If l, m, n are the direction cosines of a line then

   a. $l^2 + m^2 + n^2 = 0$
   b. $l^2 + m^2 + n^2 = 1$
   c. $l^2 + m^2 + n^2 = -1$
   d. l + m + n = 1

Ans:b

5. The distance between the parallel planes 3x + 2y – 6z + 1 =0 and 6x + 4y – 12z = -9 is

   a. 1 unit
   b. 2 units
   c. $\frac{1}{2}$ units
   d. $\frac{3}{2}$ units

Ans:c

6. If the vectors $\vec{a} = \hat{i} + 3\vec{j} - 2\vec{k}$ and $\vec{b} = -\hat{i} + 3\vec{k}$ then what is the value of $\vec{a} \times \vec{b}$?

   a. $7\hat{i} + \vec{j} + \vec{k}$
   b. $9\hat{i} + \vec{j} + 3\vec{k}$
   c. $3\hat{i} - \vec{j} + 4\vec{k}$
   d. $9\hat{i} - \vec{j} + 3\vec{k}$

Ans:d

7. If the three dice are thrown then the probability of getting exactly 2 sixes is

   a. $\frac{10}{72}$
   b. $\frac{5}{36}$
   c. $\frac{5}{72}$
   d. $\frac{10}{36}$

Ans:c

8. The derivative of $\sinh^{-1}x$ with respect to x is

   a. $\frac{1}{\sqrt{x^2 - 1}}$
   b. $\frac{1}{\sqrt{x^2 + 1}}$
   c. $\frac{1}{1 - x^2}$
   d. $\frac{1}{x\sqrt{x^2 + 1}}$

Ans:b

9. The general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$ is

   a. y = klnx
   b. y = kx
   c. lny = kx
   d. y = $\frac{k}{x}$

Ans:d

10. Which of the following operations can be performed in Gauss elimination method?

    a. $E_i \rightarrow kE_i$
    b. $E_i \leftrightarrow E_j$
    c. $E_i \rightarrow E_i + kE_j$
    d. All of the above

Ans:d

11. If a force of 100N is applied on the body for 0.1 seconds, the impulse of force is

a. 20 NS b. 10 NS
c. 100 NS d. 500 NS

Ans:b

OR

If the demand function for a monopolist is P = 50 – 2Q, for what value of Q, the total revenue is maximum?

a. Q = 25 b. Q = 12.5
c. Q = 50 d. Q = 312.5

Ans:b

Group B $5 \times 8 = 40$

12. a. Show that $1-\frac{1}{4}+\frac{1}{4}.\frac{3}{8}-\frac{1.3.5}{4.8.12}+.....to\infty=\sqrt{\frac{2}{3}}$ [3]

b. Find the value of $\frac{e+e^{-1}}{2}$.

Ans: $1+\frac{1}{2!}+\frac{1}{4!}+...$

13. Find the sum to n terms of the series $1+\frac{2}{2}+\frac{3}{2^2}+\frac{4}{2^3}+.....$

Ans: $4-\frac{1}{2^{n-2}}-\frac{n}{2^{n-1}}$

14. a. Prove that: $\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{4}{3}=\frac{\pi}{2}$ [3]

b. Show vectorically that the area of the triangle ABC whose vertices are A(1, 2, 3), B(3, 4, 5) and C(1, 4, 7) is $2\sqrt{6}$ sq. units. [2]

15. Calculate rank correlation coefficient from the data given below:

A	68	64	75	64	50	75	80	40	55	64
B	62	58	68	81	45	68	60	48	50	70

Ans:0.545

16. a. State the Rolles theorem. Verify Rolles theorem for f(x) = cos2x [0, π] [3]

b. Evaluate: $\int\frac{dx}{(\sin x+\cos x)^2}$

Ans: $-\frac{1}{\tan x+1}+c$

17. Solve: $\frac{dy}{dx}+\frac{1}{x}y=x^2y^6$

Ans:$y^5(5x^3+c_1x^5)=2$where $c_1=2c$

18. Use simplex method and maximize: P = 16x+40y subject to $3x+5y\le 30$, $x+2y\le 12$, $4x+3y\le 36$, $x,y\ge 0$

Ans:240 at (0, 6)

19. State principle of conservation of linear momentum. A gun of mass 40 metric tonnes resting on an inclined plane of 3 in 5 fires a shot of 100 kg horizontally with a velocity of 700 m/s. Find the velocity of recoil of gun and the distance it moves up the inclined before coming to rest. (g = 9.8 m/s^2) [1+4=5]

Ans:v=1.4m/s, l= $\frac{1}{6}$

OR

A firm's total cost function is given by the equation TC = 200 + 3Q, while the demand function is given by P = 107 – 2Q.

a. Write the equation of total revenue function.
b. What will be the value of Q when TR is maximum?
c. Find the equation of marginal revenue and marginal cost.

Ans:

Group C $8 \times 3 = 24$

20. A chemist produces 100 units of special drug. To produce the drug three different chemicals should be mixed up in a specific proportion i.e. the quantity of chemical. A should be one third of the difference between the chemicals B and C used, further quantity of chemical B should be twice that of C. The costs per unit of chemicals A, B and C are respectively Rs. 20, Rs. 8 and Rs. 16. If the total cost of the component chemicals is Rs. 1160, determine by using matrix method, the units of each chemical used.

Ans:10, 60, 30

21. Find the direction cosines l, m, n of two lines which are connected by equation l + m + n = 0 and 2lm + 2ln – mn = 0. Find the acute angle between two lines.

Ans: $\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}$ ***and*** $\frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}$ ***and*** $\frac{1}{\sqrt{6}}, \theta = 60^o$

22. a. Evaluate: $\int \frac{x}{x^3+1} dx$ [4]

Ans: $-\frac{1}{3}\log(x+1)+\frac{1}{6}\log\left|x^2-x+1\right|+\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)+c$

b. Solve: $(x-y)^2\frac{dy}{dx}=1$ [4]

Ans: $\frac{1}{2}\log\left(\frac{x-y-1}{x-y+1}\right)=y+c$

The End

Set – 2

Group A 1×11

1. $1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+....\infty$ is

   a. e^{-1} b. e-1
   c. e+1 d. e^0

Ans:a

2. If (G, *) is a group and $x \in G$ is any element then

   a. $x = x^{-1}$ b. $x*x^{-1}=a$
   c. $x^2 \in G$ d. $x*x^{-1}=x^{-1}$

Ans:c

3. For a numerical value of x, $\sin^{-1}\frac{1}{x} = ?$

   a. $\sec^{-1}x$ b. $\text{cosec}^{-1}x$
   c. $\sin^{-1}x$ d. $\cos^{-1}x$

Ans:b

4. The equation of the directrices of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

   a. $y = \pm\frac{b}{e}$ b. $x = \pm\frac{a}{e}$
   c. $x = \frac{2b^2}{a}$ d. $y = \frac{2a^2}{b}$

Ans:b

5. The angle between two lines whose direction ratios are 2, 3, 4 and 1, -2, 1 is

   a. $\frac{\pi}{2}$ b. $\frac{\pi}{3}$
   c. $\frac{\pi}{4}$ d. π

Ans:a

6. If $\vec{i}$, $\vec{j}$ and $\vec{k}$ are unit vectors along x axis, y-axis and z-axis respectively then the value of $\vec{i}\times\vec{i}$ is

   a. 0 b. 1
   c. -1 d. i^2

Ans:a

7. Which of the following relations is true for moderately skewed distribution?

   a. mode = median – 2 mean b. mode = 2median – 3mean

c. mode = 2(median – mean) d. mode = 3median – 2 mean

Ans:d

8. The derivative of $e^{\cos x}$ is

a. $\cos x e^{\cos x}$ b. $\cos x e^{\sin x}$

c. $-\sin x e^{\sin x}$ d. $-\sin x e^{\cos x}$

Ans:d

9. The integral of $\int \cot x dx$ is

a. $\ln|\sec x| + c$ b. $\ln|\sin x| + c$

c. $\ln|\cos x| + c$ d. $\ln|\cos ecx| + c$

Ans:b

10. The basic feasible solution of the system of equations $2x_1 - x_2 + 3x_3 = 9$ and $x_1 - x_2 + x_3 = 2$ are

a. $(0, \frac{3}{2}, \frac{7}{2})$ and (7, 5, 0) b. (0, 3, 7) and (-7, -5, 0)

c. $(0, \frac{3}{2}, \frac{7}{2})$ and (-7, -5, 0) d. $(0, -\frac{3}{2}, -\frac{7}{2})$ and (-7, -5, 0)

Ans:a

11. The greatest height attained by the projectile is given by

a. $\frac{u^2 \sin 2\alpha}{g}$ b. $\frac{u^2 \sin^2 \alpha}{2g}$

c. $\frac{u \sin \alpha}{g}$ d. $\frac{2u \sin \alpha}{g}$

Ans:b

OR

If the supply increases then the change in price

a. increases b. no change

c. becomes half d. decreases

Ans:d

Group B 5 ×8 = 40

12. a. show that the set G = {-1, 1, $-i^2$, i} the fourth roots of unity satisfies the binary operation of multiplication. [2]

b. Show that the set of integers Z forms a group under the operation of addition. [3]

13. A factory has two machines P and Q, and produced two types of articles X and Y. The following table shows the number of hours used by the two machines for the production if each unit of articles X and Y.

	P	Q
X	4	2
Y	3	1

How many maximum units of each article will be produced if the machines P and Q can work not more thatn 125 hours and 55 hours respectively in a week? And: x = 20, y = 15

Ans:x=20, y=15

14. a. Find the general solution of $2\cos^2 x + 3\sin x = 0$ [2]

Ans:x= $n\pi + (-1)^n\left(\frac{-\pi}{6}\right)$

b. Prove vectorically that sin(A-B) = sin AcosB - cosAsinB [3]

15. The data in sales and promotion expenditures on a newly launched product for 6 years are given below:

Year	2003	2004	2005	2006	2007	2008
Sales	16	20	18	24	20	22
Promotion expenses (Rs. 000)	4	4	6	10	10	12

a. Calculate the two regression coefficients from the above data of sales and expenses. [4]

b. Compute the correlation coefficient between sales and promotional expenses. [1]

16. State Lagrange's mean value theorem. Use it to find a point on the parabola $y = (x+3)^2$ where the tangent is parallel to the chord joining the points (-3, 0) and (-4, 1). [1+4]

Ans: $\left(-\frac{7}{2}, \frac{1}{4}\right)$

17. Integrate $\int \frac{x}{x^3+1}dx$ using the concept of partial fraction. Give an example of a proper rational fraction.

Ans: $-\frac{1}{3}\log|x+1| + \frac{1}{6}\log|x^2-x+1| + \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + c$

18. Use simplex method and maximize: P = 45x + 80y subject to $5x+20y \leq 400$, $10x+15y \leq 450$ and $x,y \geq 0$ ***Ans:2200 at x=24 and y=14***

19. Define parallel forces. P and Q (P>Q) are two like parallel forces acting at A and B. show that if they interchange positions, the point of application of the resultant is displaced a distance $\frac{P-Q}{P+Q}AB$. [1+4=5]

OR

The demand function for a good is given as Q = 65 – 5P. Fixed costs are Rs. 30 and each unit produces costs an additional Rs. 2.

a. Write down the equations for total revenue and total costs in terms of Q. [2]

Ans:TR=P×Q, TC=FC+VC

b. Find the break-even point algebraically. [1]

Ans:Q=2.91 and Q=52.1

c. Graph the total revenue and total costs on the same diagram.

Group C **8×3=24**

20. a. State and prove De-Moivre's theorem. [4]

b. Prove that for all $n \in N$, $(10^{2n-1}+1)$ is divisible by 11, by principle of induction. [4]

21. A line makes and angle α, β, γ and δ with the four diagonals of a cube.

a. Find the direction ratios of two diagonals. [1]

b. Find the angle between any two diagonals. ***Ans:cos$^{-1}\frac{1}{3}$***

c. Prove that: $\sin^2\alpha + \sin^2\beta + \sin^2\gamma + \sin^2\delta = \frac{8}{3}$

22. a. Evaluate: $\int \frac{dx}{(x-2)^2(x-3)^2}$ dx [4]

Ans: $-\frac{1}{2}\left(\frac{x-2}{x-3}\right)^2 + 3\left(\frac{x-2}{x-3}\right) - 3\ln\left|\frac{x-2}{x-3}\right| - \frac{x-3}{x-2} + c$

b. Solve: $\frac{dy}{dx} = \frac{x+y+1}{x+1}$ [4]

Ans: $e^{\frac{y}{x+1}} = (x+1)c$

The End

Set – 3

Group A **1 ×11=11**

1. The number of permutations of 'n' objects arranged in circle is

a. n! b. (n-1)!

c. $\frac{(n-1)!}{2}$ d. (n-2)!

Ans:b

2. The sum of first 'n' odd natural numbers is

a. $\frac{n(n+1)}{2}$ b. n(n+1)

c. n^2 d. $\frac{n(n+1)(2n+1)}{6}$

Ans:c

3. If tan-12 = A, tan-13 = B, then the value of C is equals to

a. $\frac{\pi}{3}$ b. $\frac{\pi}{4}$

c. $\frac{\pi}{2}$ d. $\frac{\pi}{6}$

Ans:b

4. The ecentricity of the hyperbola $\frac{x^2}{16} - \frac{y^2}{4} = 1$

a. $\frac{\sqrt{5}}{2}$ b. $\frac{5}{4}$

c. $\frac{2}{\sqrt{5}}$ d. $\frac{2}{5}$

Ans:a

5. The distance between the points (2, -3, 4) and (8, 3, 7) is

a. 81 units b. 9 units

c. 3 units d. $6\sqrt{2}$ units

Ans:b

6. What is the sine of angle between two vectors $\vec{a}$ = (1, 3, -2) and $\vec{b}$ = (-1, 0, 3)

a. $\frac{91}{140}$ b. $\sqrt{\frac{91}{140}}$

c. $\frac{11}{140}$ d. $\sqrt{\frac{11}{140}}$

Ans:b

7. The mean of a binomial distribution is 80 and standard deviation is 8, what is the value of p?

a. $\frac{3}{5}$ b. $\frac{4}{5}$

c. $\frac{3}{4}$ d. $\frac{1}{5}$

Ans:d

8. The equation of tangent to $y = x^3 - 2x^2 + 4$ at (2, 4) is

a. x + 4y – 18 = 0 b. $x = \pm 6$

c. 4x – y – 4 = 0 d. $y = \pm 4$

Ans:c

9. The tangent to a given curve is perpendicular to x – axis if

a. $\frac{dx}{dy} = 0$ b. $\frac{dx}{dy} = 1$

c. $\frac{dy}{dx} = 0$ d. $\frac{dy}{dx} = 1$

Ans:a

10. When Gauss Elimination method is used for solving equations x + 3y = 4 ….(i) and 2x + 5y = 9 …..(ii) we apply the operation like

a. eqn I + eqn ii b. 3eqn I – eqn ii

c. eqn I + 3eqn ii d. eqn ii + 3eqn i

Ans:b

11. The pull of the earth on a body is 49N. If the acceleration due to gravity is g = 9.8 m/s^2, the mass of the body is

a. 5 kg b. 4 kg

c. 49 kg d. 98 kg

Ans:a

OR

Which equation for total revenue of the followings is true?

a. TR=P×Q b. TR=P+Q

c. TR=P-Q d. TR=FC+VC

Ans:a

Group B 5 ×8 = 40

12. Prove that: $\frac{5}{1.2.3} + \frac{7}{3.4.5} + \frac{9}{5.6.7} + ...to\infty = -1 + 3\log 2$

13. For the given equation $(k-2)x^2+(k-5)x-5=0$ when $k \neq 2$, find k so that

    a. the roots are equal ***Ans: -5*** $\pm 2\sqrt{10}$

    b. the roots are numerically equal but opposite in sign. ***Ans: 5***

14. a. If $\cos^{-1}x + \cos^{-1}y+\cos^{-1}z = \pi$ show that $x^2 + y^2 + z^2 + 2xyz = 1$ [3]

    b. Find the area of the parallelogram whose diagonals are determined by the vectors $\vec{a} = 3\vec{i} + \vec{j} - 2\vec{k}$ and $\vec{b} = \vec{i} - 3\vec{j} + 4\vec{k}$ [2]

    Ans: $5\sqrt{3}$

15. Given the bivariate data:

X	1	5	3	2	1	1	7	3
Y	6	1	0	0	1	2	1	5

    Find the regression equation of y on x and hence estimate y when x = 8.

    Ans:y=2.875+(-0.304)x when x=8, y=0.443

16. Find the derivative of $(\tan x)^{\cot x} + (\cot x)^{\tan x}$ [1+4]

    Ans: $cosec^2x(\tan x)^{\cot x}[1-\ln(\tan x)]+sec^2x(\cot x)^{\tan x}.[\ln(\cot x)-1]$

17. a. Evaluate $\int \sqrt{\frac{1+x}{1-x}}dx$ ***Ans:*** $\sin^{-1}x - \sqrt{1-x^2} + c$

    b. Evaluate using partial fraction. $\int \frac{x}{(x+1)(x+2)}dx$

    Ans: $-\ln(x+1)+2\ln(x+2)+c$

18. Use simplex method and maximize: $Z = 9x_1 + x_2$ subject to $2x_1+x_2 \leq 8$, $4x_1+x_2 \leq 8$ and $x_1, x_2 \geq 0$

    Ans:36 at $x_1=4, x_2=0$

19. A projectile thrown from a point in a horizontal plane comes back to the plane in 4 secs at a distance of 60m in from of the point of projection, find the velocity of projection. ($g=10m/s^2$) [1+4=5]

    Ans:25

OR

Solve non-homogeneous first order difference equations

a. Find the general solution of the difference equation $y_{t+1} - 0.85y_t = 1000$ [2]

    Ans: $y_t=A(0.95)^t+20000$

b. Find the particular solution, given $y_5 = 20950$. [1]

    Ans: $y_t=1228(0.95)^t+20000$

Group C **8×3=24**

20. Dairy corporation is planning to produce 1200 kg of baby food in a week mixing three nutrients A, B and C. The three nutrients components per kg costs Rs. 12, Rs. 16 and Rs. 14 respectively. The final output of 1 kg baby food pack must contain nutrient C to be twice that of nutrient A. The weekly budget available to buy component nutrients to the corporation is Rs. 18480. Find the amount of each nutrient which should be mixed in the final output. Use row equivalent matrix method. ***Ans: 90, 930, 180***

21. a. Deduce the equation of a hyperbola with a focus at (6, 0) and a vertex (4, 0). ***Ans:*** $\frac{x^2}{16} - \frac{y^2}{20} = 1$

    b. Find the angle between the two lines whose direction ratios are a_1, b_1, c_1 and a_2, b_2, c_2. Also find the condition under which the two lines are perpendicular.

    Ans: $\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$, $a_1a_2+b_1b_2+c_1c_2=0$

22. a. State Rolles theorem. What is the geometrical intrepretaion of Rolles theorem. Verify Rolles theorem for $f(x) = \sin x + \cos x$ on $[0, 2\pi]$ [4]

    b. Solve: $\frac{dy}{dx} + \frac{1}{x}y = x^2$ [4]

    Ans: $xy = \frac{x^4}{4} + c$

Set – 4

Group A **1 ×11=11**

1. The total number of three digit numbers that can be formed using the integers 1, 2, 3 and 4 is

   a. 10 b. 24

   c. 42 d. 64

Ans:b

2. If 1, ω and ω^2 are the cube roots of unity then which one of following is true?

   a. $1+\omega+\omega^2=0$ b. $1+\omega+\omega^2=1$

   c. $1+\omega+\omega^2=-1$ d. $1+\omega+\omega^2=\omega^3$

Ans:a

3. The principle value off $\cos^{-1}\left(\frac{1}{2}\right)$ is

   a. $\frac{\pi}{3}$ b. $\frac{\pi}{4}$

   c. $\frac{\pi}{2}$ d. $\frac{\pi}{6}$

Ans:a

4. The length of perpendicular drawn from (2, 3, -5) to the plane x+ 2y -2z = 9 is

   a. 1 b. 2

   c. 3 d. 4

Ans:c

5. The cosine angle between two diagonals of a cube is

   a. $\frac{1}{2}$ b. $\frac{1}{3}$

   c. $\frac{1}{4}$ d. $\frac{1}{5}$

Ans:b

6. If $\left|\vec{a}\times\vec{b}\right|=\left|\vec{a}.\vec{b}\right|$ then angle between the vector $\vec{a}$ and $\vec{b}$ is

   a. π b. $\frac{\pi}{2}$

   c. $\frac{\pi}{3}$ d. $\frac{\pi}{4}$

Ans:d

7. If A and B are two events and P(A) = 0.5 and P(B)=0.6 and $P(A\cup B)=0.8$ then value of P(A/B) is

   a. $\frac{1}{2}$ b. $\frac{3}{5}$

   c. $\frac{3}{10}$ d. $\frac{2}{5}$

Ans:a

8. The limit of function $\left(\frac{x-\sin x}{x^3}\right)$ at x=0 is

   a. $\frac{1}{3}$ b. $\frac{4}{5}$

   c. $\frac{2}{3}$ d. $\frac{1}{6}$

Ans:d

9. The integral of $\int\frac{1}{a^2+x^2}dx$ is

   a. $\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c$ b. $\frac{1}{a}\sin^{-1}\left(\frac{x}{a}\right)+c$

   c. $\frac{1}{a}\cos^{-1}\left(\frac{x}{a}\right)+c$ d. None of the above

Ans:a

10. The solution of linear systems x + 3y = 4 and 2x + 5y = 9 is

a. (7, 1) b. (7, -1)
c. (-7, -1) d. (-7, 1)

Ans:b

11. A motorcyclist increases its velocity at the rate of 5m/s to 30m/s in seconds. What is its initial velocity?

a. 5 m/s b. 10m/s
c. 20m/s d. 30m/s

Ans:b

OR

The demand function for an item is P = 65 – 5Q. which of the followings denote the total revenue function??

a. 65 – 5PQ b. $65 – 5Q^2$
c. $65Q – 5Q^2$ d. 65Q – 5PQ

Ans:c

Group B **5 ×8 = 40**

12. State Binomial theorem. In the expansion of $(1+x)^n$ prove that the sum of the coefficients of the odd terms is equal to the sum of coefficients of the even terms and each equals to 2^{n-1}. [5]

13. Prove that: $1.3+2.4+3.5+....n(n+2)=\frac{n(n+1)(2n+7)}{6}$ by the method of induction.

14. a. If $\cos^{-1}x + \cos^{-1}y+\cos^{-1}z= \pi$, show that $x^2+y^2+z^2+2xyz=1$ [3]

b. If $\vec{a} = \hat{i} +3\hat{j} -2\hat{k}$ and $\vec{b} = -\hat{i}+3\hat{k}$, Find

i. $\vec{a} \times \vec{b}$

ii. a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$

Ans:i. $9\hat{i}-\hat{j}+3\hat{k}$ ***ii.*** $\frac{9}{\sqrt{91}}\hat{i}-\frac{1}{\sqrt{91}}\hat{j}+\frac{3}{\sqrt{91}}\hat{k}$

15. From the following pair of regression equations, find the correlation coefficient between the two variables x and y.

4x - 5y + 33 = 0 and 20x - 9y - 107 = 0, interpret the result. ***Ans:r=0.6***

16. Using mean value theorem, find the point on the curve $f(x) = x^2 – 2x$, the tangent at which is parallel to the chord joining the points (1, -1) and (4, 8). [1+4]

Ans: $\left(\frac{5}{2},\frac{5}{4}\right)$

17. Prove that $\int \frac{dx}{\sqrt{a^2-x^2}}dx = \sin^{-1}\frac{x}{a}+c$

b. Evaluate. $\int \frac{dx}{2+\cos x}$

Ans: $\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{\tan\frac{x}{2}}{\sqrt{3}}\right)$

18. What are the two steps of Gauss elimination method? Using Gauss elimination method solve the following linear systems:

2x+y-3z=5, x+y-z=6, x-3y+3z=-6

Ans: 3, 5, 2

19. A ball is thrown upwards at an angle of 30^o to the horizon and lands on the top edge of a building that is $10\sqrt{3}$ m away. The top edge is 5m above the throwing point. How fast the ball thrown? [1+4=5]

Ans: 19.8m/s

OR

Calculate the consumer surplus for the demand function P = 60 – 2Q when the market price P_o = 12 by integration. Also calculate the producer surplus at Q = 4 when P = 20 + 4Q by integration. [1]

Ans: CS = 576, PS = 32

Group C **8×3=24**

20. a. Solve the following equation using row operation

X – 3y – 7z = 6, 2x + 3y +1 = 9 and 4x + y = 7 [4]

Ans: x=1, y=3, z=-2

b. Show that S = $\left\{.....,\frac{1}{2^2},\frac{1}{2},1,2,2^2,2^3,....\right\}$ forms an abelian group under multiplication.

21. Find the equation of the plane through the intersection of the planes x + y + z = 6 and 2x +3y 4z +5 = 0 and is perpendicular to the plane 4x + 5y – 3z = 8. Also find the distance of the plane from 2x + 14y + 26z + 100 = 0

Ans:x+7y+13z+96=0

22. a. Find the derivative of $(\sin x)^{\tan x} + (\cos x)^{\sec x}$ with respect to x.[4]

Ans: $(\sin x)^{\tan x}.(1+\sec^2 x \log_e \sin x)+(\cos x)^{\sec x}.\sec x \tan x(\log_e \cos x-1)$

b. Solve: $\frac{dy}{dx}=\frac{x^2+y^2}{2x^2}$ [4]

Ans:2x=(x-y)lncx

Set – 5

Group A **1 ×11=11**

1. The sum of two roots of quadratic equation $Ax^2+Bx+C=0$

a. $\frac{C}{B}$ b. $\frac{C}{A}$

c. $\frac{-B}{A}$ d. $\frac{B}{A}$

Ans:c

2. The coefficient of x^3 in the expansion of e^{2x+3}

a. 1 b. 2

c. $\frac{2}{3}$ d. $\frac{4}{3}$

Ans:d

3. The value of $\cos\sin^{-1}\frac{3}{5}$ is

a. $\frac{3}{5}$ b. $\frac{4}{5}$

c. $\frac{5}{4}$ d. $\frac{5}{3}$

Ans:b

4. Angle between two planes x+2y+z+7=0 and 2x+y-z+13=0 is

a. $\frac{\pi}{2}$ b. $\frac{\pi}{3}$

c. $\frac{\pi}{4}$ d. $\frac{\pi}{6}$

Ans:b

5. If a line makes the angles A, B and C with the axes respectively then Cos2A+Cos2B+Cos2C equals

a. -1 b. 0

c. 1 d. none

Ans:a

6. The value of λ if $3\vec{i}+\vec{j}-\vec{k}$ and $\lambda\vec{i}-4\vec{j}+4\vec{k}$ are collinear vectors is

a. -12 b. 12

c. 10 d. 15

Ans:a

7. Find the standard deviation if p = $\frac{1}{3}$

a. $\frac{2}{3}$ b. $\frac{8}{3}$

c. $\frac{4}{3}$ d. $\frac{1}{6}$

Ans:c

8. Slope of tangent of $y= -3x-x^4$ at x=-1 is

a. 1 b. 2
c. 3 d. 4

Ans:a

9. The value of $\int (a^2 - x^2)^{\frac{1}{2}} dx$ is

a. $\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)+c$ b. $\frac{x}{2}\sqrt{x^2-a^2}+\sin^{-1}\left(\frac{x}{a}\right)+c$

c. $\frac{x^2}{2}\sqrt{x^2-a^2}+\frac{a^2}{2}\log(x+\sqrt{x^2-a^2})+c$ d. $\frac{x}{2}\sqrt{x^2-a^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)+c$

Ans:d

10. The solution set of $4x+3 \geq 2x-1$ is

a. $x \leq -2$ b. $x \geq 2$
c. $x \geq -2$ d. $x \leq 2$

Ans:c

11. If one of two like parallel forces and their resultant are 18N and 36N respectively. The ratio of distances of resultant from the forces.

a. 1:2 b. 1:3
c. 1:4 d. 1:1

Ans:d

OR

The break-even occurs when

a. TR<TC b. TR>C
c. TR=TC d. TR=2TC

Ans:c

Group B $5 \times 8 = 40$

12. Show that the set of all vectors in space under addition is a group.

13. Show that : $\sum_{N=1}^{\infty} \frac{n2}{(n+1)!} = e-1$

14. a. Evaluate: $\text{costan}^{-1}\text{sincot}^{-1}\left(\frac{4}{3}\right)$ ***Ans:*** $\frac{5}{\sqrt{34}}$

b. Find the area of ΔABC whose vertices are A(1, 2, 3) B(2, 5, -1) and C(-1, 1, 2) vectorically. ***Ans:*** $\frac{1}{2}\sqrt{155}$

15. The edquations of two regressions lines are 3X + 4Y = 65, 3X + Y = 32. Find

i. the mean of X and the mean of Y [2]
ii. regressions coefficeints [2]
iii. correlation coefficeints between X and Y. [1]

16. Find from the first principles the derivative of $\log_e(\cos\frac{x}{a})$.

Ans: $-\frac{1}{a}\tan\frac{x}{a}$

17. Integrate using the concept of partial fraction. $\int \frac{x2}{x^4 - 2x^2 - 15} dx$

Ans: $\frac{\sqrt{3}}{8}\tan -1\frac{x}{\sqrt{3}} + \frac{\sqrt{5}}{6}\log\frac{x-\sqrt{5}}{x+\sqrt{5}} + c$

18. Using simplex method maximize U = 25x + 45y subject to $x+3y \leq 21$, $2x+3y \leq 24$, $x,y \geq 0$ ***Ans:U=345 at x=3 and y=6***

19. A canon ball has the same range R on the horizontal plane for two different angles of projection. If H and H' are the greatest heights in two paths for which this is possible, prove that $R^2 = 15HH'$

Ans: 19.8m/s

OR

If the demand function $P_D = Q^2 - 10Q + 25$ and supply function is $P_S = Q^2 + 6Q + 9$, sketch the graph of each function over the interval Q = 0 to Q = 5. Also find the equilibrium price and quantity.

Ans: Q=1, P=16

Group C **8×3=24**

20. a. If the roots of $lx^2 + nx+n=0$ be in the ratio p:q, prove that:

$$\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0 \qquad [4]$$

b. An automobile manufacturer uses three different types of trucks T_1, T_2 and T_3 to transport motorbikes cars and jeeps as shown in the following table.

Trucks	Motorbikes	Cars	jeeps
T_1	2	3	6
T_2	6	7	6
T_3	9	12	8

Using matrix method to determine the number of trucks of each type required to supply 116 motorbikes, 150 cars and 124 jeeps to a dealer.

Ans:T_1 = 4, T_2 = 6, T_3 = 8

21. The two straight lines have direction ratios a_1, b_1, c_1 and a_2, b_2, c_2. Write the condition for being the lines parallel and perpendicular. Show that the straight lines whose direction cosines are given by the equations al + bm + cn = 0 and ul^2, $vm^2 + wn^2 = 0$ are perpendicular if $a^2(v+w) + b^2(u+w) + c^2(u+v) = 0$ and parallel if $\frac{a^2}{u} + \frac{b^2}{v} + \frac{c^2}{w} = 0$

Ans: $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ ***, $a_1a_2+b_1b_2+c_1c_2=0$***

22. a. State Rolles theorem. Verify that the function f(x) = x(x-3)2 on [0, 3] satisfies conditions of Rolles theorem and find c prescribed in theorem.

b. Solve: $\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x}$ [4]

Ans: $\sin\frac{y}{x} = cx$

The End