Ignore the question numbers

Limit and Continuity

(a) $\lim\limits_{x\to 2}[2(x+3)+7]$ (b) $\lim\limits_{x\to 0}\left(x^2+3x+7\right)$ (c) $\lim\limits_{x\to 1}\left[(x+3)^2-16\right]$

(d) $\lim\limits_{x\to -1}\left[(x+1)^2+2\right]$ (e) $\lim\limits_{x\to 0}\left[(2x+1)^3-5\right]$ (f) $\lim\limits_{x\to 1}(3x+1)(x+1)$

2. Find the limits of each of the following functions :

(a) $\lim\limits_{x\to 5}\frac{x-5}{x+2}$ (b) $\lim\limits_{x\to 1}\frac{x+2}{x+1}$ (c) $\lim\limits_{x\to -1}\frac{3x+5}{x-10}$

(d) $\lim\limits_{x\to 0}\frac{px+q}{ax+b}$ (e) $\lim\limits_{x\to 3}\frac{x^2-9}{x-3}$ (f) $\lim\limits_{x\to -5}\frac{x^2-25}{x+5}$

(g) $\lim\limits_{x\to 2}\frac{x^2-x-2}{x^2-3x+2}$ (h) $\lim\limits_{x\to \frac{1}{3}}\frac{9x^2-1}{3x-1}$

3. Evaluate each of the following limits:

(a) $\lim\limits_{x\to 1}\frac{x^3-1}{x-1}$ (b) $\lim\limits_{x\to 0}\frac{x^3+7x}{x^2+2x}$ (c) $\lim\limits_{x\to 1}\frac{x^4-1}{x-1}$

(d) $\lim\limits_{x\to 1}\left[\frac{1}{x-1}-\frac{2}{x^2-1}\right]$

4. Evaluate each of the following limits :

(a) $\lim\limits_{x\to 0}\frac{\sqrt{4+x}-\sqrt{4-x}}{x}$ (b) $\lim\limits_{x\to 0}\frac{\sqrt{2+x}-\sqrt{2}}{x}$ (c) $\lim\limits_{x\to 3}\frac{\sqrt{3+x}-\sqrt{6}}{x-3}$

(d) $\lim\limits_{x\to 0}\frac{x}{\sqrt{1+x}-1}$ (e) $\lim\limits_{x\to 2}\frac{\sqrt{3x-2}-x}{2-\sqrt{6-x}}$

8. Find the value of 'a' such that $\lim\limits_{x\to 2}f(x)$ exists, where $f(x)=\begin{cases}ax+5, & x<2\\ x-1, & x\geq 2\end{cases}$

9. Let $f(x)=\begin{cases}x, & x<1\\ 1, & x=1\\ x^2, & x>1\end{cases}$

Establish the existence of $\lim\limits_{x\to 1}f(x)$.

10. Find $\lim\limits_{x\to 2}f(x)$ if it exists, where

$$f(x)=\begin{cases}x-1, & x<2\\ 1, & x=2\\ x+1, & x>2\end{cases}$$

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(b) If $f(x)=\begin{cases}4x+3, & x\neq 2\\ 3x+5, & x=2\end{cases}$, find whether the function f is continuous at $x=2$.

(c) Determine whether $f(x)$ is continuous at $x=2$, where

$$f(x)=\begin{cases}4x+3, & x\leq 2\\ 8-x, & x>2\end{cases}$$

(d) Examine the continuity of $f(x)$ at $x=1$, where

$$f(x)=\begin{cases}x^2, & x\leq 1\\ x+5, & x>1\end{cases}$$

(e) Determine the values of k so that the function

$$f(x)=\begin{cases}kx^2, & x\leq 2\\ 3, & x>2\end{cases}$$ is con tinuous at $x=2$.

(b) Test the continuity of the function $f(x)$ at $x=1$, where

$$f(x)=\begin{cases}\dfrac{x^2-4x+3}{x-1} & \text{for } x\neq 1\\ -2 & \text{for } x=1\end{cases}$$

(c) For what value of k is the following function continuous at $x=1$?

$$f(x)=\begin{cases}\dfrac{x^2-1}{x-1} & \text{when } x\neq 1\\ k & \text{when } x=1\end{cases}$$

(d) Discuss the continuity of the function $f(x)$ at $x=2$, when

$$f(x)=\begin{cases}\dfrac{x^2-4}{x-2}, & \text{for } x\neq 2\\ 7, & x=2\end{cases}$$

Determine the points of discontinuity, if any, of the following functions :

(a) $\dfrac{x^2+3}{x^2+x+1}$ (b) $\dfrac{4x^2+3x+5}{x^2-2x+1}$

(b) $\dfrac{x^2+x+1}{x^2-3x+1}$ (d) $f(x)=\begin{cases}x^4-16, & x\neq 2\\ 16, & x=2\end{cases}$

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Solution for finding points of discontinuity

Find the point of discontinuity for the following function:

$$p(x)=\frac{x^3-4x^2-5x}{x^2-9x+20}$$

Explanation:

Start by factoring the numerator and denominator of the function.

$$p(x)=\frac{x^3-4x^2-5x}{x^2-9x+20}=\frac{x(x^2-4x-5)}{(x-5)(x-4)}=\frac{x(x-5)(x+1)}{(x-5)(x-4)}=\frac{x(x+1)}{x-4}$$

A point of discontinuity occurs when a number a is both a zero of the numerator and denominator.

Since $x=5$ is a zero for both the numerator and denominator, there is a point of discontinuity there. To find the y value, plug in 5 into the final simplified equation.

$$\frac{5(5+1)}{5-4}=30$$

$(5,30)$ is the point of discontinuity.

Find a point of discontinuity in the following function:

$$f(x)=\frac{x^2+x-12}{x^2+2x-8}$$

Explanation:

Start by factoring the numerator and denominator of the function.

$$f(x)=\frac{x^2+x-12}{x^2+2x-8}=\frac{(x+4)(x-3)}{(x+4)(x-2)}=\frac{(x-3)}{(x-2)}$$

A point of discontinuity occurs when a number a is both a zero of the numerator and denominator.

Since $x=-4$ is a zero for both the numerator and denominator, there is a point of discontinuity there. To find the y value, plug in -4 into the final simplified equation.

$$\frac{-4-3}{-4-2}=\frac{7}{6}$$

$\left(-4,\frac{7}{6}\right)$ is the point of discontinuity.